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Frequency Accuracy & Stability Dependencies of Crystal Oscillators

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Abstract

Quartz crystal based oscillators are used as clock sources in the synchronization and syntonization of distributed systems to a common time or frequency scale. One such system is that of a cellular network in which base station transceivers are operated within a specified time or frequency accuracy with reference to a system reference. The accuracy of the entrainment of the distributed clocks to the reference clock is subject to the design of the servo control system. In the event the servo fails the slave clock accuracy is a function of the local environmental and electrical stimuli applied to the clock. As loss of the servo signal is a practical issue in a real system, this ultimate system entrainment accuracy is dependent on the accuracy with which the free running clocks can be corrected. It is the subject of the current paper to review the fundamental physical properties of crystal oscillators and in so doing determine all significant frequency perturbing stimuli. Identification and quantification of these stimuli in terms of analytical expressions is the first stage in the creation of an accurate clock model suitable for compensation of the clock in the absence of the servo signal from the reference. Thus a fundamental understanding of the parameters affecting the clock drift becomes paramount to determining the overall synchronization accuracy achievable by the system.

1. Introduction

Time is very important not only for the daily schedules of human beings, but also for processing of the sequence of events that happens in computers, and for time-tagging information that flows through communication systems. So the clock sources are essential for almost all electronic equipments and communication systems. Clock sources (another name is frequency control devices) can provide precise time and frequency on which modern electronic equipments depend. If all frequency control devices stop working, all modern communication systems (telephones, radios, TV stations, air traffic control systems, etc.) would stop functioning, all transportation systems (automobiles, trucks, airplanes) would cease operating, and all computers would stop. [11]

In the modern world, a vibrating quartz crystal is the heart of nearly all frequency control devices. Quartz crystal oscillators provide accurate time and are the sources of precise frequency, which are electronic circuits that use the mechanical resonances of vibrating crystals of piezoelectric materials to create periodically varying electrical signals. The frequency stability, cost and size of quartz crystal oscillators has resulted in their ubiquitous usage as a frequency reference in electronic equipment. Crystal oscillators as frequency sources and frequency control components are most widely used in the time and frequency research and production fields, such as the IT industry, Communications, Electronic Instruments, Applied Electronic Techniques, Measurements, Aerospace systems, Radar, Military Industry, etc... In the modern world, a quartz crystal oscillator is the only option for a not too expensive but reasonably precise and stable frequency source. Although some other materials like ceramic resonators have been developed, their frequency stability and accuracy cannot compare with quartz crystals. According to different accuracy, stability and cost requirements, different types of crystal oscillators are employed. The temperature dependence of the crystal resonance is a generally recognized first order perturbation to the frequency accuracy of the crystal oscillator. Compensation of the temperature dependence has resulted in a classification of crystal oscillators based on the different temperature control methods, like SPXO (Simple Packaged Crystal Oscillator), which has no temperature compensation; TCXO (Temperature Compensation Crystal Oscillator), which uses analog or digital temperature compensation circuits; OCXO (Oven Controlled Crystal Oscillator), which uses an oven to control crystal temperature; and DOCXO (Double Oven Controlled Crystal Oscillator), which uses two temperature control ovens, one inside the other, to further improve the stabilization of the crystal temperature relative to variations in the ambient temperature.

2. Crystal Resonator

The crystal resonator is the most important component of a crystal oscillator and the quartz crystal is the “heart” of it. A quartz crystal is an anisotropic crystal formed from silicon dioxide. The crystal structure consists of two pyramidal ends and is hexagonal in cross-section. Figure 1 illustrates the physical structure of a quartz crystal.

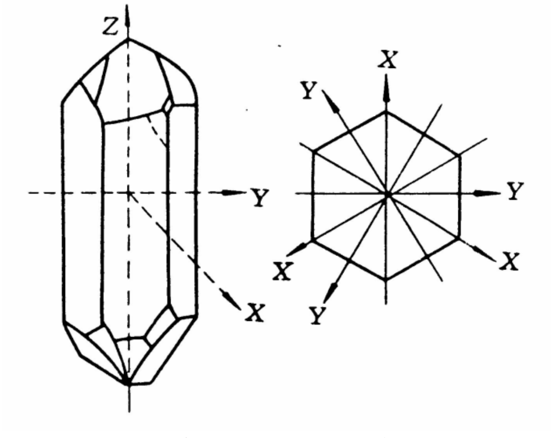


Figure 1. Crystal

Here, the Z axis is the optical axis, the X axis is the electric axis, and the Y axis is the mechanical axis. A quartz crystal exhibits a piezoelectric effect. When force is applied to either the Y axis or the X axis, then the two surfaces which are vertical to this axis will have opposite charges, and the value is directly proportional to the lattice deformation caused by the mechanical pressure. On the other hand, if an electrical field is applied on opposite surfaces of the crystal, according to different electrical field directions, the crystal will stretch or compress in proportion to the applied electrical field strength. The crystal is cleaved along a particular crystal plane to achieve a particular electromechanical characteristic. Typically crystal cuts used in quartz oscillators are AT-cut and SC-cut. Electrodes are plated on two surfaces of the crystal and then the crystal is encapsulated in a metal or glass enclosure. The enclosure is either evacuated or filled with an inert gas.

The frequency of a crystal resonator is determined by the cut, vibration mode, and the size of the crystal wafer. If, for example, a longitudinal vibration mode is excited, the resonant frequency is approximately based on the equation below.

$$f_o = 2.7\times 10^3 / L \qquad (1)$$

L is the crystal size parameter, unit is meter and the numerical constant represents the phase velocity of the vibration in the crystal. So, if f_0 is 100 kHz, L should be 2.7cm. If f_0 is 10 MHz, L should be 0.27 mm. Because processing very small crystals is difficult, the oscillator circuit can be designed to excite the crystal in an overtone mode. Use of a larger crystal also has the added advantage that it reduces the sensitivity of the oscillator to mechanical vibration. A typical overtone crystal oscillator works at 3 times, 5 times or 7 times of the fundamental crystal resonance frequency.

3. Physical and electrical factors affecting crystal oscillator frequency stability and accuracy

The frequency accuracy of a crystal oscillator is the offset from the specified target frequency. The frequency stability of the oscillator is the spread of the measured oscillator frequency about its operational frequency in a period time. Figure 2 shows the accuracy and stability examples for a frequency source. Factors such as temperature, crystal aging and retrace establish the frequency accuracy of the oscillator, whereas reference signal noise (if the oscillator is locked to a reference), tuning port noise, supply rail noise, and vibration establish the stability of the oscillator. With respect to applications reliant on synchronization, random frequency perturbations of zero mean are less significant compared to the frequency accuracy of the oscillator. The dependence of synchronization on oscillator frequency accuracy is because time error is the integral of the frequency error. In the case of syntonization, the frequency stability must be contained within specification but there is no cumulative error over time resulting from a static frequency error within the frequency bounds of the system specification.

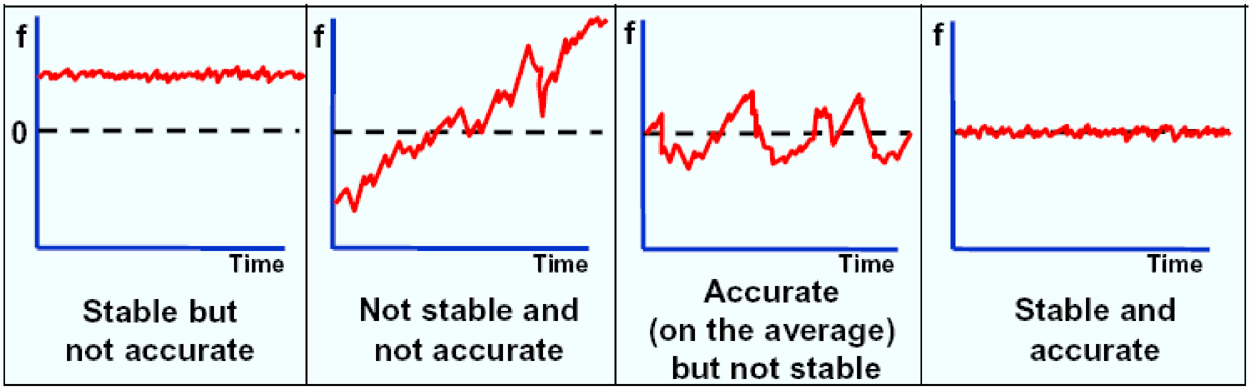


Figure 2. Accuracy and Stability examples for a frequency source [3]

3.1 The factors affecting crystal oscillator frequency accuracy

3.1.1 Temperature

Temperature is a significant factor which affects the frequency of resonators. Different crystal cuts have a different frequency-temperature characteristic. Figure 3 shows the frequency-temperature property of a typical AT-cut crystal resonator (Here, AT, SC, or GT represent different crystal cut methods). The φ represents cut angle. We can see that the crystals with different cut angles have different frequency-temperature curves. Below are some crystal resonator temperature characteristics.

1) The crystal cuts in general exhibit a cubic dependence on temperature [3].

2) In most situations, the zero temperature coefficient point can be changed through changing the angle between crystal wafer and crystal axis.

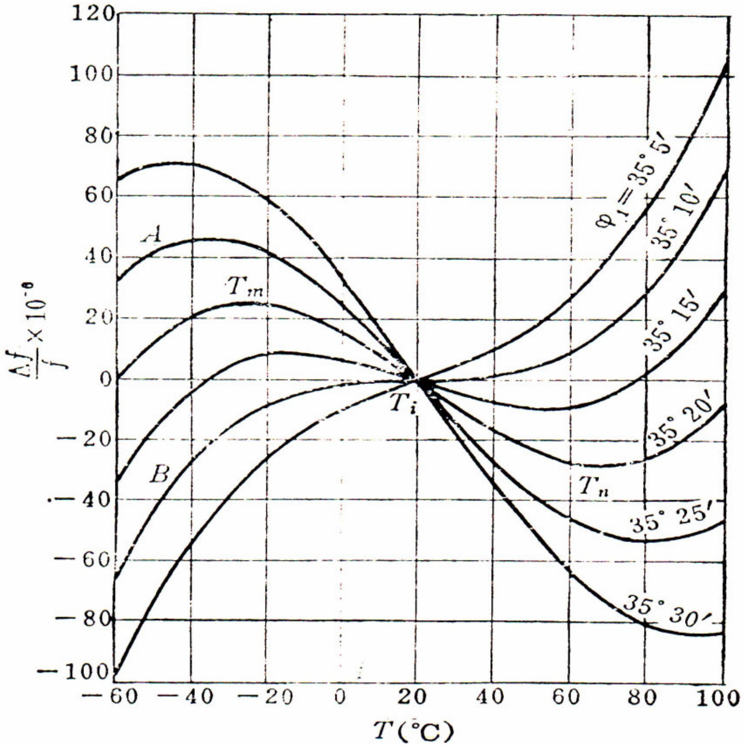


Figure 3. AT-cut crystal resonator frequency-temperature property [4]

3) In a wide temperature range, like -55~+105℃， the relative frequency change of AT and GT cut crystals can be limited to $\pm 2\times10^{-5}$ with a suitable angle processing.

3.1.2 Aging

The crystal resonator frequency will change according to the operational time and this physical phenomenon is termed aging. A representative aging plot is shown in Figure 4.

Figure 4. Aging of Crystal Resonator [3]

It should be noted that although the plot is monotonic, this is not always the case and the aging rate can reverse sign over time. When the vibration mode of a crystal wafer is Thickness-Shear, as in AT cut and SC cut crystals, aging mostly results from:

1) Thermal gradient effects. It will continue several minutes to several hours after thermal equilibrium [4]. Figure 5 shows the temperature gradient effects and warm-up characteristics of two OCXOs, each containing an oven which reaches the thermal equilibrium in six minutes (referring to [11], chapter "Warm-Up", about warm-up property of oscillators). One contains an AT-cut and the other contains an SC-cut. The frequency variation after six minutes comes from thermal gradient effects in the figure 5. We can see SC-cut OCXO has much better performance than AT-cut OCXO. We do not need to consider aging rate before thermal equilibrium, because it only takes 3 to 10 minutes for OCXO and a few seconds for other oscillators.

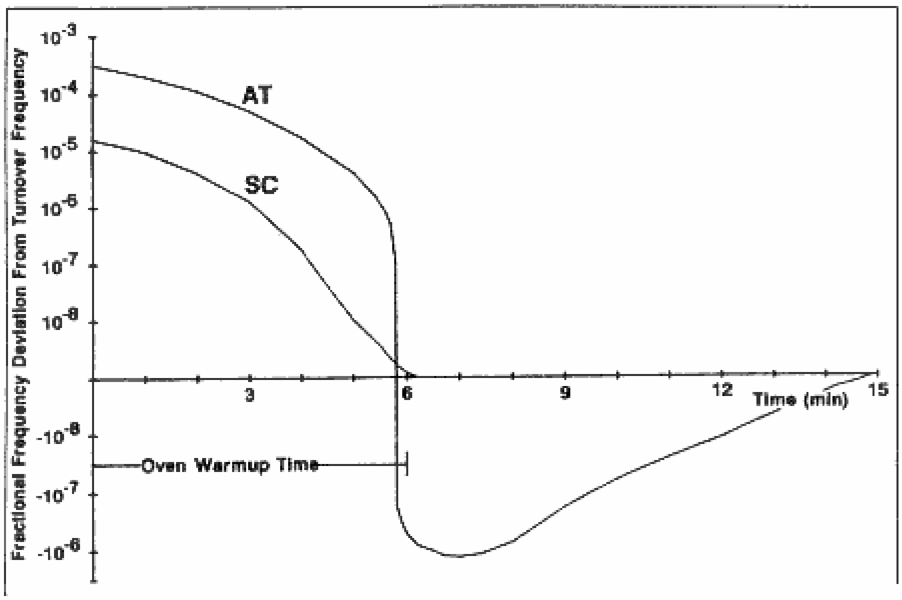


Figure 5. Warm-up characteristics and thermal gradient effects of AT-cut and SC-cut crystal oscillators (OCXOs)[11]

2) Pressure release effect. It is a function of the heat process above, and will continue 3 days to 3 months. [4]
3) The increase and decrease of the crystal polar plates mass, which is caused by gas absorption and decomposition, and will continue several weeks to several years. [4]
4) Crystal structure change caused by a defective crystal lattice, which is a long-term effect.

In low-frequency quartz crystal resonators, when the vibration mode is face-shear, the aging rate is the lowest. The aging rate is higher in the case of bending vibration and the extension vibration results in the highest aging rate. When the vibration mode is the same, then a lower frequency and a bigger polar plate crystal experience a lower aging rate. Aging effects can be divided into two time periods, the prior period and the later period. The prior period aging (for 1 to 2 months) has a higher aging rate and this aging rate can reach 1×10^{-7}/month (the number means frequency accuracy will change 1×10^{-7} per month) to 1×10^{-8}/month. For the later period, when a crystal has been operational for 1~2 months, the aging rate can reduce to $(1\sim3)\times10^{-9}$/month to $(1\sim3)\times10^{-10}$/month.

3.1.3 Drive Level

In a precise crystal resonator, the oscillator frequency also relies on the crystal electric current or drive level. The equation is:

$$\Delta f / f \approx ki^2 \qquad (2)$$

Here, i is the alternating-current which flows through the crystal; k is a constant which depends on the crystals. $\Delta f / f$ is the relative variation of the vibration frequency. When the drive electric current is big, the aging property and long-term frequency stability will be worse. But when the drive level is too small, the noise electric current may be relatively bigger than crystal electric current, and this will cause the worse short-term frequency stability. Currently, the normal 2.5 MHz and 5 MHz high-precision crystal oscillators driving level is less than $70\,\mu A$ [4]. The current high quality SC-cut crystal oscillator has perfect solution to overcome the drive level effect. The methods of measurement of drive level dependence in use today include: 1) Test oscillators [10]. 2) Passive measurements using IEC-444 pi-network with vector voltmeter and phase locked signal generator, complemented with variable attenuators and preamplifier. 3) Passive network analyzer measurements at fixed frequency and swept output level.

3.1.4 Retrace

When power is removed from an oscillator for several hours, then re-applied on it again, the frequency of this oscillator will stabilize at a slightly different value. This frequency variation error is called retrace error. It is usually occurring for twenty four or more hours off-time followed by a warm-up time which is enough to complete thermal equilibrium. Retrace errors will reduce after warming. The shape of the error curve is like that the crystal walks back down its aging curve when cold and then moves toward the prior drift curve when activated. If the resonator is in its aging phase, the retrace error will be added to the aging drift, while with well-aged resonators the frequency will look for a new level characteristic for alternating operation. Usually, retrace errors show clearly less spread with SC cut than with AT cut resonators. Careful selection of crystals, oscillators can decrease the influence from retrace effect which is as close as a few parts in 10^{10} [6]. Retrace is one of factors that affect frequency accuracy of OCXO. For TCXO or other oscillators, retrace is usually not considered as a significant affecting factor to frequency accuracy [11].

Figure 6 shows how OCXO retrace influences oscillator frequency accuracy. The x axis represents time and the y axis represents frequency accuracy. In (a), the oscillator was kept on continuously while the oven was cycled off and on. In (b), the oven was kept on continuously while the oscillator was cycled off and on.

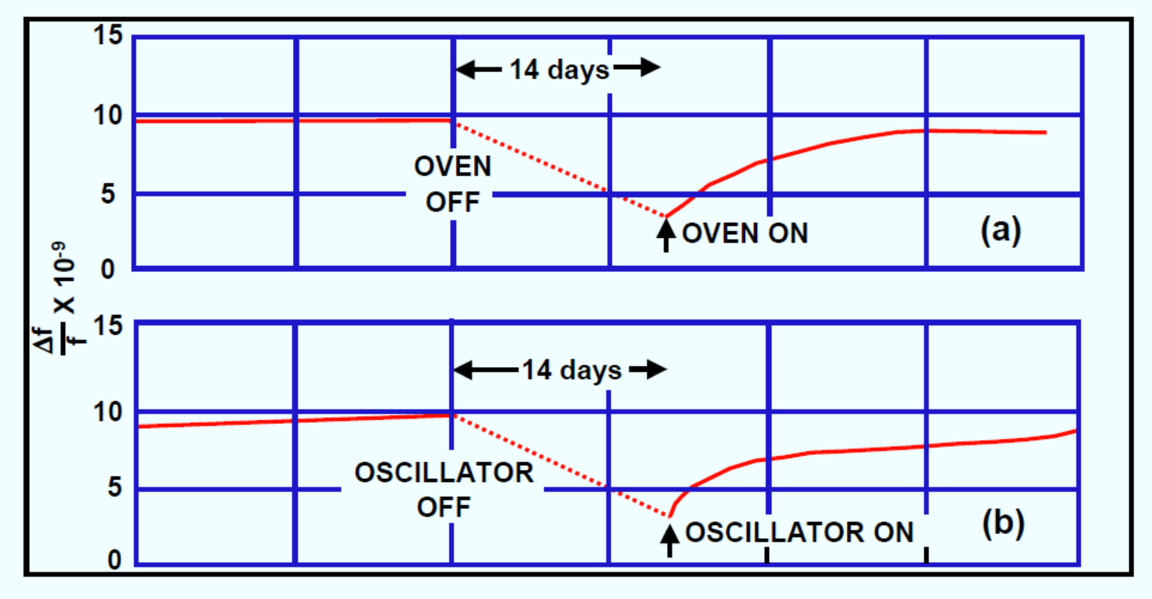


Figure 6. OCXO Retrace [11]

3.1.5 Thermal Hysteresis

The frequency-temperature characteristic of a quartz crystal resonator does not duplicate exactly upon temperature cycling. This effect is called thermal hysteresis. The "hysteresis" is defined as the difference between the up-cycle and the down-cycle frequency vs. temperature characteristics and is quantified by the value of the difference at the temperature where that difference is maximal [5]. Thermal Hysteresis effect applies to all crystal based oscillators to a greater or lesser extent it has been investigated in TCXO [11].

The research results indicate that lattice defects are related to thermal hysteresis. Stress relief in the mounting structure can also create considerable hysteresis. The parts in 10^9 hysteresis have been observed in some resonators [7].

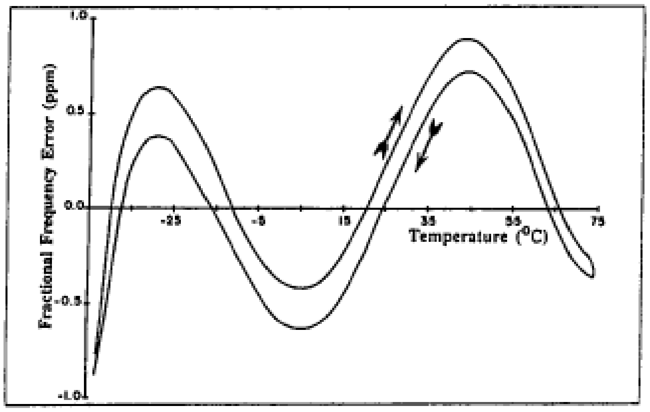


Figure 7. TCXO Thermal Hysteresis [7]

Figure 7 shows that the first frequency - temperature characteristic upon increasing temperature differs from the characteristic upon decreasing temperature.

3.1.6 Frequency Pushing and Frequency Pulling

Frequency pushing is a measure of the sensitivity of the oscillator output frequency to supply voltage, which is expressed in MHz/volt. Frequency Pulling is a measure of the frequency change because of a non-ideal load. Both of them affect the oscillator circuitry and indirectly the drive level of resonator and load reactance. The change in load impedance modifies the phase or amplitude of the signal reflected into the oscillator loop, which changes the frequency of the oscillators. The effects can be diminished by using a buffer amplifier and a low noise voltage regulator [8]. Frequency pushing and frequency pulling are not important factors for high frequency accuracy oscillators because technologies have already been used for solving these problems. They are not in the specifications of these oscillators. So for choosing high frequency accuracy oscillators (such as OCXOs with frequency accuracy of several ppb) as system components, frequency pushing and frequency pulling may not be considered.

3.1.7 Tuning port reference voltage drift

The tuning port reference voltage drift will also affect the accuracy of frequency of oscillator because the tuning range and tuning sensitivity is utterly dependent on the tuning voltage. The jitter or drift of the voltage will cause an inaccurate frequency regulation. This factor is relative to the tuning voltage sensitivity of the oscillators and it has different extent influence on different kinds of oscillators.

3.2 Factors affecting frequency stability

3.2.1 Oscillator tuning port noise

Tuning-port noise can affect the stability of oscillators by affecting tuning sensitivity. Tuning sensitivity is a system-level parameter that relates the maximum available tuning voltage to the required tuning-frequency range, in units of Hz/volt. If the tuning sensitivity of a VCXO (voltage controlled crystal oscillator) varies dramatically over the tuning band, the performance of oscillator in a phase locked loop will be worse as the noise bandwidth of the control loop will vary as a function of VCXO frequency. The tuning sensitivity can change in response to noise at the tuning port. So tuning-port noise must be minimized.

3.2.2 Reference source noise

When a reference frequency source is applied to a crystal oscillator, the reference source noise will also affect the stability of the oscillator. The reference frequency source noise contributes to the overall phase noise. Phase noise is a big topic in crystal oscillator design and development. It will affect short-term stability of oscillators.

3.2.3 Power supply noise

The power supply noise is one of the sources of oscillator phase noise. Especially for ring-based voltage-controlled oscillators, it is the dominant noise source. Such noise typically appears as steps or impulses on the power supply of the oscillator, and it affects both frequency and phase, causing cycle-to-cycle jitter [9].

3.2.4 Vibration-induced noise

Vibration-induced noise is another source of phase noise. It is caused by the sensitivity to acceleration of crystals. So the random and periodic mechanical vibrations found in many types of equipment and instruments can induce significant phase noise in high-performance crystal oscillators. It can be classified to sine vibration-induced phase noise and random vibration-induced phase noise [2].

3.2.5 Others

Other factors affecting oscillator stability include electric field, magnetic field, ambient pressure (altitude), humidity, acceleration, gas permeation, etc [2].

3.3 Factors Comparison

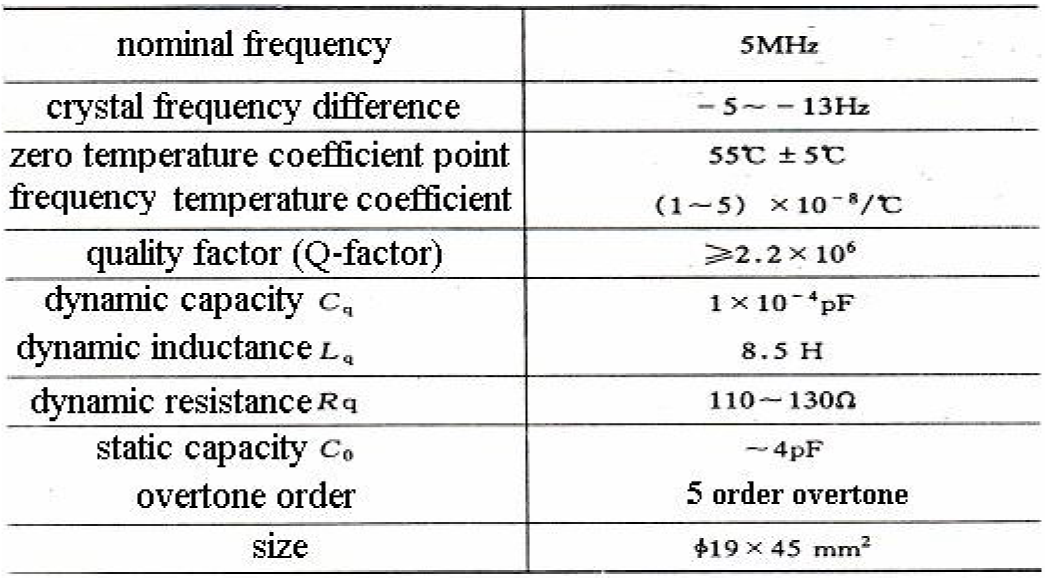
Different influence factors which affect accuracy and stability of crystal oscillators have different weights depending on the operating conditions of the oscillator. Temperature and aging drift are the most important factors which affects the accuracy of oscillators. In the case that the thermal environment is stable the aging induced frequency error may dominate the frequency behavior of the oscillator. Alternatively if the thermal environment is undergoing variations in a time frame that is short in comparison with the time required for the oscillator to drift significantly with respect to the aging rate of the crystal then the temperature dependent frequency stability of the oscillator will dominate the temporal stability of the clock. Ranking of other factors is highly dependent on the working environment of the oscillator and as such must be done on a case by case basis.

Various kinds of noises are factors which affect the stability of oscillators. There are no documents to compare which noise is the dominant factor, and this is still based on the specific application.

For example, we consider an OCXO used in wireless station which references the GPS signal to keep its frequency accuracy. When the GPS reference signal is lost, the OCXO will enter “holdover” state and its accuracy will become worse. Normal operation of the base transceiver station, the maximal cumulative time error of the OCXO in a period of time (such as 24 hours) will be limited and the OCXO will be enhanced by voltage control circuitry. In this situation, we should focus on the factors which affect the frequency accuracy of OCXO because the short term frequency stability will not contribute to the cumulative time error. The temperature and aging are two dominant factors. If the OCXO does not need to be frequently turned on and off, the retrace effect will be minor. And using high quality SC-cut crystal oscillator will eliminate the effect of the drive level. Thermal hysteresis impacts the temporal stability of the oscillator and must be considered if the temporal error introduced by the hysteresis is significant with respect to the required accuracy of the clock. Frequency pushing and frequency pulling are also minor factors. But since the voltage control tuning port is introduced in the OCXO, we should limit the tuning port reference voltage drift.

4. Parameters of quartz crystal resonators

As the most important component of the crystal oscillator, quartz crystal resonators have many technical parameters which show their property. List 1 gives the characteristic parameters of typical 5 MHz precise quartz crystal resonator.



nominal frequency	5MHz
crystal frequency difference	$-5 \sim -13$Hz
zero temperature coefficient point	55℃ ± 5℃
frequency temperature coefficient	$(1 \sim 5) \times 10^{-8}$/℃
quality factor (Q-factor)	$\geqslant 2.2 \times 10^{6}$
dynamic capacity C_q	1×10^{-4}pF
dynamic inductance L_q	8.5 H
dynamic resistance R_q	110 ~ 130Ω
static capacity C_0	~4pF
overtone order	5 order overtone
size	$\phi 19 \times 45$ mm^2

Table 1. Parameters for 5 MHz crystal resonator [4]

Here, the nominal frequency is the frequency that the quartz crystal resonator is designed to work at. The zero temperature coefficient point means the temperature in which the frequency – temperature coefficient reaches the minimum. The crystal frequency difference means the difference between working frequency and crystal series resonance frequency when the resonator is working around the zero temperature coefficient point. The frequency temperature coefficient here means resonator frequency accuracy for every degree variation, when crystal works around the zero temperature coefficient point (e.g. 50 degree to 60 degree here). The quality factor (Q-factor) means the ratio of the frequency at which the resonator works and the rate at which it dissipates its energy. A higher Q-factor indicates a lower rate of energy dissipation relative to the oscillation frequency, so the oscillations die out more slowly. Dynamic capacity, dynamic inductance, and dynamic resistance are equivalent capacity, equivalent inductance, and equivalent resistance of the resonators when resonators are working, which are useful for circuit analysis. Static capacity is the capacity value of the resonator when the resonator is not working, which is used in energy storage analysis of resonators. The overtone order means the resonator works on overtone mode and the times of fundamental mode frequency.

5. Quartz Crystal Oscillator types

5.1 SPXO (Simple Packaged Crystal Oscillator)

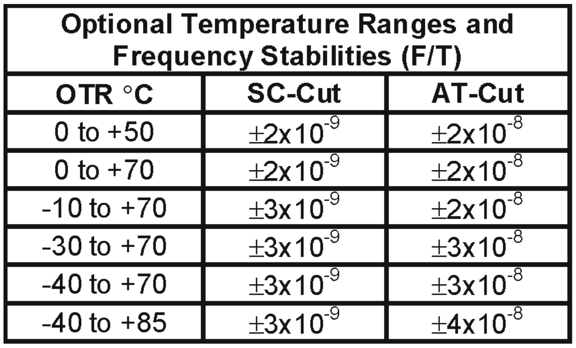
SPXOs are used mostly widely. They are used in many different electrical devices such as computers as clock signal source. They only include main oscillation circuits and output circuits. SPXOs do not take any measure to eliminate the temperature affection on vibration frequency. They can reach a frequency stability of $10^{-4} \sim 10^{-5}$ which is the lowest in the crystal oscillator family, but they also give the lowest price.

5.1 TCXO

TCXOs can compensate the temperature perturbation of the crystal frequency. They can achieve frequency stability of $10^{-6} \sim 10^{-7}$. They can include analog, digital, and microprocessor compensation. Microprocessor compensation crystal oscillator (MCXO) can reach a stability of 10^{-8} .

5.2 OCXO

Most highly stable crystal oscillators use a thermostatical control oven to guarantee high stability. In a thermostatic control oven, the temperature is tuned to the zero temperature coefficient point. List 2 shows typical data for an MtronPTI's XO5120 as an example to show the frequency stability over temperature achievable using OCXO technology.

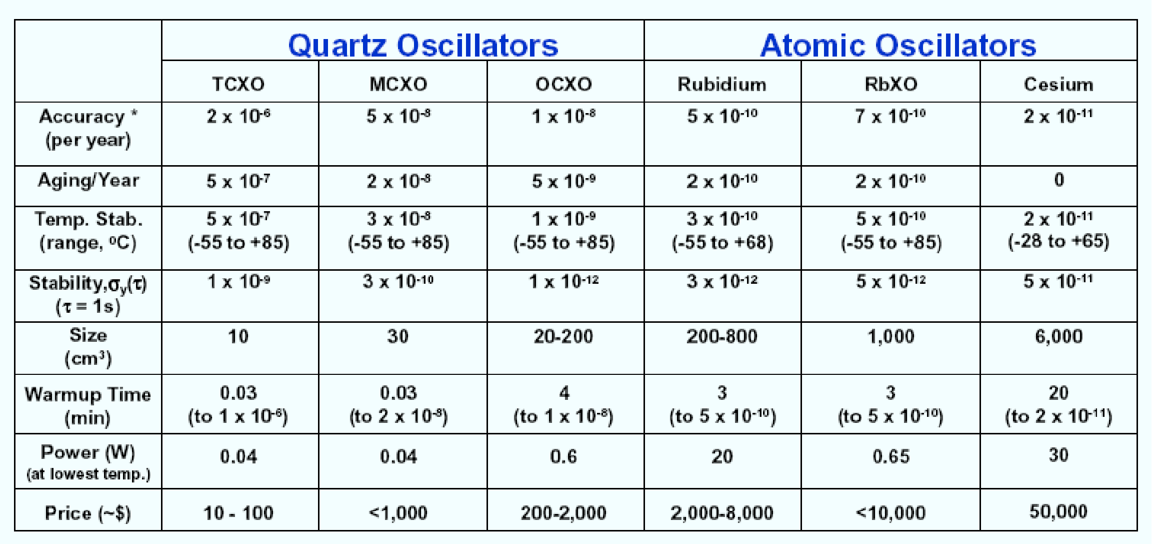


Optional Temperature Ranges and Frequency Stabilities (F/T)		
OTR °C	**SC-Cut**	**AT-Cut**
0 to +50	$\pm 2x10^{-9}$	$\pm 2x10^{-8}$
0 to +70	$\pm 2x10^{-9}$	$\pm 2x10^{-8}$
-10 to +70	$\pm 3x10^{-9}$	$\pm 2x10^{-8}$
-30 to +70	$\pm 3x10^{-9}$	$\pm 3x10^{-8}$
-40 to +70	$\pm 3x10^{-9}$	$\pm 3x10^{-8}$
-40 to +85	$\pm 3x10^{-9}$	$\pm 4x10^{-8}$

Table 2. Temperature Ranges and their Frequency Stability of MtronPTI's XO5120 [1]

6. Other Clocks

Besides crystal oscillators, other kinds of clocks are being used. There are different kinds of atomic clocks, like Cesium atomic clocks, Rubidium gas cell atomic clocks, and the Hydrogen maser frequency standard. Typically they are more precise and more stable than crystal oscillators, and they are also more expensive. List 3 demonstrates the differences among different crystal oscillators and atomic clocks.



	Quartz Oscillators			Atomic Oscillators		
	TCXO	MCXO	OCXO	Rubidium	RbXO	Cesium
Accuracy * (per year)	2×10^{-6}	5×10^{-8}	1×10^{-8}	5×10^{-10}	7×10^{-10}	2×10^{-11}
Aging/Year	5×10^{-7}	2×10^{-8}	5×10^{-9}	2×10^{-10}	2×10^{-10}	0
Temp. Stab. (range, ^oC)	5×10^{-7} (-55 to +85)	3×10^{-8} (-55 to +85)	1×10^{-9} (-55 to +85)	3×10^{-10} (-55 to +68)	5×10^{-10} (-55 to +85)	2×10^{-11} (-28 to +65)
Stability, $\sigma_y(\tau)$ ($\tau = 1s$)	1×10^{-9}	3×10^{-10}	1×10^{-12}	3×10^{-12}	5×10^{-12}	5×10^{-11}
Size (cm^3)	10	30	20-200	200-800	1,000	6,000
Warmup Time (min)	0.03 (to 1×10^{-6})	0.03 (to 2×10^{-8})	4 (to 1×10^{-8})	3 (to 5×10^{-10})	3 (to 5×10^{-10})	20 (to 2×10^{-11})
Power (W) (at lowest temp.)	0.04	0.04	0.6	20	0.65	30
Price (~$)	10 - 100	<1,000	200-2,000	2,000-8,000	<10,000	50,000

Table 3. Comparison among Quartz Oscillators and Atomic Oscillators [2]

Sometimes when high precision and stability is not needed, an electronic oscillator can be used, such as RC oscillators (Resistors and Capacitors oscillators), which only contain simple resistor and capacitor oscillation circuit or LC oscillators (Inductors and Capacitors oscillators), which only contain simple inductor and capacitor oscillation circuit. But their precision and stability are much worse than crystal oscillators, and can only be used in environments with very low precision requirements.

7. Conclusion

This paper is a summary paper about frequency accuracy and stability dependency of crystal oscillators. The crystal resonator is the "heart" of the crystal oscillator and the crystal oscillator is the most important clock source nowadays. Many factors that influence the frequency accuracy and stability of the crystal oscillators are reviewed. Some typical parameters of crystal resonators are given. The paper also lists the typical types of crystal oscillators. In the end, a specification comparison including the prices among different types of crystal oscillators and atomic clocks is given.

References:

[1]. XO5120 specification on OCXOs products list from www.mtronpti.com

[2]. John R Vig, "Introduction to quartz frequency standards" Technical Report SLCET-TR-92-1, Army Research Laboratory, Electronics and Power Sources Directorate,1992.[Online],available:http://www.ieeeuffc.org/freqcontrol/quartz/vig/vigtoc.htm. March 19, 2006

[3]. John R. Vig, “Quartz Crystal Resonators and Oscillators for Frequency Control and Timing Applications - A Tutorial”, 2004 IEEE International Frequency Control Symposium Tutorials, May 2004.

[4]. Wei Zhou, “Time, Frequency measurement and control technology”, Xidian University Press, 2006.

[5]. U.S. Department of Defense, Military Specification, Oscillators, Crystal, Genera1 Specification for MIL-PRF-55310. Online available at
http://www.dscc.dla.mil/Programs/MilSpec/ListDocs.asp?BasicDoc=MIL-PRF-55310

[6]. Euler, F; Yannoni, N.F, “Frequency Retrace of Quartz Oscillators”, Proceedings of 35th Annual Frequency Control Symposium. 1981

[7]. John A. Kusters; John R. Vig, “Thermal Hysteresis in Quartz Resonators – A Review”, Proceedings of 45th Annual Symposium on Frequency Control. 1990

[8]. Mini-Circuits notes on VCO test methods from www.minicircuits.com

[9]. T.Pialis and K. Phang, “Analysis of timing Jitter in Ring Oscillators Due to Power Supply Noise”, Circuits and Systems, ISCAS’03, Proceedings of the 2003 International Symposium. 2003

[10]. Yerna, Y. C, “Resistance measurements at very low drive levels”, Proceedings of 38th Annual Frequency Control Symposium. 1984

[11]. J. Vig and A. Ballato, “Ultrasonic Instruments and Devices”, Academic Press Inc. pp. 637–701 (Chapter7: Frequency Control Devices). 1999